International Conference on Control, Engineering & Information Technology (CEIT'13)
Proceedings Engineering & Technology - Vol.3, pp. 117-122, 2013

Design of fuzzy sliding mode observers for Anaerobic digestion process

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***Abstract*—Fuzzy modelling and state estimation/observation are powerful methods for the control and diagnosis of very complex uncertain systems as bioprocesses. Especially where the system's states are not all available for measurement because hardware sensor measurements does not exist. This paper presents, a fuzzy representation and robust state estimation of anaerobic digestion system using fuzzy sliding mode observers (FSMO), the stability of the FSMO is established.**

Keyword: *T-S fuzzy modelling, state estimation, Sliding mode, Anaerobic digester, LMI***.**

I. Introduction

Modern wastewater treatment plants (WWTP) contribute greatly in reducing the impact of human waste on the environment. This is accomplished by removing most of the oxygen demand caused by chemical and organic wastes in the wastewater prior to its return to the environment [6]. While there is a big variety in processes and equipment used at WWTPs, the basic processes are common to most large plants.
Anaerobic digestion is a waste treatment technology which presents a number of advantages. It allows for the stabilization of the waste through the controlled destruction of the organic material to produce an end product (digestate) that can be used as a fertilizer or soil conditioner and it produces renewable energy.

Moreover, anaerobic digestion plant (ADP) has great potential as a renewable energy source. The process produces a biogas, comprising of methane and carbon dioxide. This biogas can be used directly as cooking fuel, in combined heat and power gas engines or upgraded to natural gas quality biomethane. The utilisation of biogas as a fuel helps to replace fossil fuels.

The bioprocesses are complexes, nonlinear and uncertain systems. They are in most of the time, badly definite, and submitted to unexpected changes.

It is more useful to represent nonlinear complex systems by local linear time invariant (LTI) models obtained by a multiple operating point linearization. Fuzzy systems have the capacity to handle in the same framework numeric and linguistic information. This characteristic made these systems very useful to handle expert control tasks [5].

Takagi-Sugeno modelling is a very interesting representation of nonlinear systems because they permit to represent most of nonlinear systems, whatever is its complexity, by a simple structure. The representation of the nonlinear systems introduced in [4] constitutes an interesting alternative in the domain of control, observation and diagnosis of nonlinear systems.

Anaerobic digestion variables are not all available for measurement because hardware sensor measurements does not exist wich are very important for control and diagnosis [3]. In this work, we used a sliding mode observers as soft sensors to estimate all variables of ADP.

Sliding mode observers (SMO) and unknown input observers [16] [17], are a developed type of observer based method. The structure of SMOs is the key of their robustness and insensitivity to the various types of uncertainties [12], [11]. These observers are more strong than Luenberger observer because they offer advantages inherent robustness by taking in consideration the modelling uncertainties, faults, perturbations or/and non linearities in the plant [10].

To ensure the convergence of the state estimation error, an LMI based design procedures for different types of observers are constructed in [8], [9],[17], [14] and [7].

In this paper, we investigate the use of fuzzy sliding mode observer for anaerobic digestion system state estimation . The rest of this paper is organized as follows: Section II describes briefly the analytic anaerobic digestion model. Section III outlines the fuzzy modelling. Section IV outlines the T-S fuzzy sliding mode observer design and convergence conditions of the FSMO presented in terms of linear matrix inequality (LMI) formulation. Experimental results are presented in section V. Finally, some concluding remarks as well as some possible improvements are given in section VI.

II. ANAEROBIC DIGESTION SYSTEM DESCRIPTION

Anaerobic digestion is a multi-step process by which organic solids are degraded and converted to methane CH_4 and carbon dioxide CO_2, commonly referred to collectively as biogas. The organic solids consist of proteins, carbohydrates, and lipids, which are converted to biogas through three sequential, metabolic stages shown in fig.1: 1.) hydrolysis and fermentation, 2.) volatile fatty acid (VFA) oxidation and 3.) biogas formation, There are several models available for the simulation of the anaerobic digestion of wastewater solids and sludges. Some of these models focus on a specific component of anaerobic digestion, such as microbial kinetics, while others attempt to encompass the overall digestion process [1],[2].

$$\begin{cases} \dot{X}_1 = (\mu_1 - \alpha D)X_1 \\ \dot{X}_2 = (\mu_2 - \alpha D)X_2 \\ \dot{S}_1 = D(S_1^{in} - S_1) - k_1\mu_1 X_1 \\ \dot{S}_2 = D(S_2^{in} - S_2) - k_2\mu_1 X_1 - k_3\mu_2 X_2 \\ \dot{Z} = D(Z^{in} - Z) \\ \dot{C}_{TI} = D(C_{TI}^{in} - C_{TI}) + k_7(k_8 P_{co_2} + \\ Z - C_{TI} - S_2) + K_4\mu_1 X_1 + k_5\mu_2 X_2 \\ Q_{CH_4} = k_6\mu_2 X_2 \end{cases} \tag{1}$$

Where $X_1, X_2, S_1, S_2, Z, C_{TI}$ are respectively, the bacteria concentration acidogenic $[g/L]$, methanogenic bacteria $[g/L]$, soluble COD $[g/L]$, total volatile fatty acids VFA $[mmol/L]$, total inorganic carbon and total alkalinity $[meq/L]$. For these variables $"in"$ indicates the influent concentration. D is the dilution rate and Q_{CH_4} is the biogas (methane) flow rate $[L/h]$.
Nonlinear functions μ_1 and μ_2 represent the Haldane growth rates and have the following structure :

$$\mu_1 = \mu_{\max 1}\frac{S_1}{k_{s_1} + S_1} \; and \; \mu_2 = \mu_0\frac{S_2}{k_{s2} + S_2 + \left(\frac{S_2}{k_{I2}}\right)^2} \tag{2}$$

where $k_1, ..., k_8$ are the yield coefficients.
The partial pressure P_{CO_2} of CO_2 have the following structure :

$$P_{CO_2} = \frac{\Phi - \sqrt{\phi^2 - 4k_8 P_r\,[CO_2]}}{2k_8} \tag{3}$$

Where P_r is total pressure in the reactor $[atm]$,
$\Phi = k_8 P_r + [CO_2] + \frac{k_6\mu_2 X_2}{k_1}$ and $[CO_2] = C_{TI} + S_2 - Z$

III. FUZZY MODELLING

A. Linearization of the process model

Many dynamical nonlinear uncertain systemes as (NUS) (1) can be described by differential equations :

$$\begin{cases} \dot{x}(t) = f(x,u) \\ y(t) = h(x,u) \end{cases} \tag{4}$$

Where f and h are nonlinear functions.
It is better to represent nonlinear systems by local LTI models (7) obtained by a multiple operating point linearization. Fuzzy systems have the capacity to handle in the same framework numeric and linguistic information. The obtained linearized model corresponds to the relationship between the variation of the system output and the variation of the system input around this operating point. If the system is linearized around an operating point $(\bar{x}_i, \bar{u}_i)$, the linearized model corresponds to the relationship between the variations of the system states x and inputs u such that :

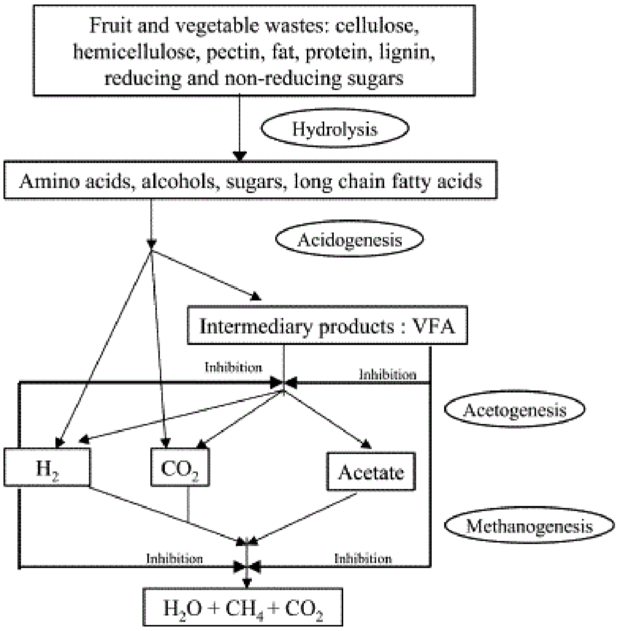

Figure 1. The general schematic diagram of anaerobic digestion process.

$$x = x - \bar{x}_i \quad and \quad u = u - \bar{u}_i \tag{5}$$

$$A_i = \left.\frac{\partial f}{\partial x}\right|_{(\bar{x}_i,\bar{u}_i)} \quad B_i = \left.\frac{\partial f}{\partial x}\right|_{(\bar{x}_i,\bar{u}_i)} \quad C_i = \left.\frac{\partial h}{\partial x}\right|_{(\bar{x}_i,\bar{u}_i)} \tag{6}$$

From (6), we obtain the following local state-space description :

$$\begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) + a_i \\ y = C_i x(t) + c_i \end{cases} \tag{7}$$

Let's note that $(\bar{x}_i, \bar{u}_i)$ is not necessarily an equilibrium point, what means that constants must be gotten for each sub-system i :

$$\begin{cases} a_i = f(x_i, u_i) - A_i\bar{x}_i - B_i\bar{u}_i \\ c_i = h(x_i, u_i) - C_i\bar{x}_i \end{cases} \tag{8}$$

It is worth noticing that the local state space model can be identified from experimental data [18].

B. T-S Fuzzy model

Fuzzy systems are used to simplify the complexity of strong nonlinear systems. This representation belongs to the paradigm of behavioral representation in opposition to the structural representation (neural networks). The foundation of this paradigm is that intelligent behavior can be obtained by the use of the structures that not necessarily resemble the human brain. The model of the plant is assumed to be given by the T-S models, where the i-th rule is of the form :

Rule i :

$IF\ \ z_1(t)\ \ is\ \ M_{1i}\ \ AND\ ...\ AND\ \ z_g(t)\ \ is\ M_{gi}$

THEN $\begin{cases} \dot{x} = A_i x(t) + B_i u(t) + E_i d(t) + a_i \\ y(t) = C_i x(t) + c_i \end{cases}$

where $x \in \Re^n$ is the state vector, $u \in \Re^m$ is the control input vector, $y \in \Re^p$ is the system output vector and $d \in \Re^q$ contains unknown inputs and it is assumed to be bounded (9). $z(t) = \{z_1(t), ..., z_g(t)\}^T$ is the decision vector, M_{gi} are membership functions, $A_i \in \Re^{n\times n}, B_i \in \Re^{n\times m}, C_i \in \Re^{p\times n}, E_i \in \Re^{n\times q}$ are known matrices and $i = 1, ..., r$ with r is the number of rules.

$$\|d(t)\| \leq \rho \tag{9}$$

The whole dynamic system defined by T-S fuzzy system is given by :

$$\begin{cases} \dot{x}(t) = \sum\limits_{i=1}^{r} h_i(z(t)) \left[A_i x(t) + B_i u(t) + E_i d(t) + a_i\right] \\ y(t) = \sum\limits_{i=1}^{r} h_i(z(t)) \left[C_i x(t) + c_i\right] \end{cases} \tag{10}$$

Where $h_i(z(t)) = \frac{\mu_i(z(t))}{\sum\limits_{i=1}^{r} \mu_i(z(t))}$

$\mu_i(z(t)) = \prod\limits_{j=1}^{g} \mu_{ij}(z_j(t))$ is the degree of fulfilment of the i-th rule, $\mu_{ij}(z_j(t))$ is the grade of membership function of $z_j(t)$ in M_{ij}, and M_{ij} is a fuzzy set.

$h_i(z(t)))$ must satisfy the following constraints :

$$\begin{cases} \sum\limits_{i=1}^{r} h_i(z(t)) = 1, \\ h_i(z(t)) \in [0,1], \quad \forall i = 1, ..., r. \end{cases} \tag{11}$$

Such that, B_i is full rank, the pairs (A_i, B_i) are completely controllable and the pairs (A_i, C_i) are completely observable.

IV. Fuzzy sliding mode observer (FSMO) design

Sliding mode observer offer advantages inherent robustness by taking in consideration the modelling uncertainties, perturbations or/and non linearities in the plant [14]. The FSMO for the system (10) by r rules has the following form :

Rule i :

$IF\ \ z_1(t)\ \ is\ \ M_{1i}\ \ AND\ ...\ AND\ \ z_g(t)\ \ is\ M_{gi}$

Then

$\begin{cases} \hat{x}(t) = A_i \hat{x}(t) + B_i u(t) + L_i e_y(t) + \varphi_i(t) \\ \hat{y}(t) = C_i \hat{x}(t) \end{cases}$

The whole FSMO design is :

$$\begin{cases} \dot{\hat{x}}(t) = \sum\limits_{i=1}^{r} h_i(z(t)) \left[A_i \hat{x}(t) + B_i u(t) + L_i e_y(t) + \varphi_i(t)\right] \\ \hat{y}(t) = \sum\limits_{i=1}^{r} h_i(z(t)) C_i \hat{x}(t) \end{cases} \tag{12}$$

Where $h_i(z(t))$ are the same as the weights in the T-S fuzzy model (10), $L_i \in \Re^{n\times p}$ is the observer gain matrix and $\varphi_i \in \Re^m$ is an external discontinuous vector of sliding mode observer to compensate the errors due the unknown inputs, such that $C_i L_i$ is a non singular matrix.

The aim of the observer design task is to find appropriate gain matrices L_i and variable vectors that guarantee the asymptotic convergence of $\hat{x}(t)$ toward $x(t)$.

To analyze the convergence of the SMO, the state reconstruction error is defined as follows :

$$e_x(t) = x(t) - \hat{x}(t) \tag{13}$$

and the output reconstruction error is :

$$e_y(t) = y(t) - \hat{y}(t) = \sum_{i=1}^{r} \mu_i(z(t)) (C_i e_x(t)) \tag{14}$$

Using (13) and (14) we got :

$$\begin{array}{l} \dot{e}_x(t) = \sum\limits_{i=1}^{r} \sum\limits_{j=1}^{r} h_i(z(t)) h_j(z(t)) \\ (\bar{A}_{ij} e(t) + E_i d(t) - \varphi_i(t)) \end{array} \tag{15}$$

Where $\bar{A}_{ij} = A_i - K_i C_j$ for $(i, j = 1, .., r)$ and $i < j$. The asymptotic convergence of the estimation error is expressed in the following result :

Theorem 1 : Let us consider the T-S fuzzy model (10), if there exist symmetric and positive definite matrices $P \in \Re^{n\times n}$ and $N_i \in \Re^{n\times p}$, positive scalars ε_1, ε_2 and γ, the observer's synthesis takes place by the resolution of a set of linear matrix inequalities LMIs and structural constraints given by :

$$\begin{bmatrix} A_i P + P A_i - C_j^T N_i^T - N_i C_j + \gamma I & P \\ P & -\varepsilon_1 I \end{bmatrix} < 0 \tag{16}$$

Then, if the equations $\varphi_i(t)$ are given by the following form :

$$\varphi_i(t) \equiv \begin{cases} \rho^2 \varepsilon_2^{-1} \frac{\|PE_i\|^2}{2 e_y^T e_y} P^{-1} \sum\limits_{j=1}^{r} h_j C_j^T e_y & if \quad e_y(t) \neq 0 \\ 0 & otherwise \end{cases} \tag{17}$$

The fuzzy sliding mode observer (12) with $L_i = P^{-1} N_i$, guarantees the asymptotic convergence to zero of the estimation error.

Lemma 1: for any matrices X and Y with appropriate dimensions, the following property holds for any positive scalar β :

$X^T Y + Y^T X \leq \beta X^T X + \frac{1}{\beta} Y^T Y$

Proof: Considering the quadratic Lyapunov function :

$$V(e_x) = e_x^T P e_x \tag{18}$$

Its time derivative $\dot{V}(t) = e_x^T P \dot{e}_x + \dot{e}_x^T P e_x$ using (13), (15) and (16) has the fom :

$$\dot{V} = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(z) h_j(z) \left(e_x^T \left(\bar{A}_{ij}^T P + P \bar{A}_{ij} \right) e_x + \right.$$

$2e_x^T PE_i d - 2e_x^T P\varphi_i)$ (19)

Using Lemma 1 the derivative of the lyapunov function can be written :

$$\dot{V} \leq \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z)h_j(z)\left(e_x^T(\bar{A}_{ij}^T P + P\bar{A}_{ij})e_x + \varepsilon_1^{-1} e_x^T P^2 e_x + 2e_x^T PE_i d - 2e_x^T P\varphi_i\right)$$
$$\leq \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z)h_j(z)\left(e_x^T\left(\bar{A}_{ij}^T P + P\bar{A}_{ij} + \varepsilon_1^{-1}P^2\right)e_x + 2e_x^T PE_i d - 2e_x^T P\varphi_i\right)$$

Where $\forall i,j \in \{1,...,r\} / h_i(z(t))\, h_j(z(t)) \neq 0$

Applying Lemma 1 for each term of $\dot{V}$ we got :

$$2e_x^T PE_i d = e_x^T PE_i d + d^T E_i^T P e_x$$
$$2e_x^T PE_i d \leq \varepsilon_2 e_x^T e_x + \varepsilon_2^{-1}\left\|PE_i d\right\|^2$$
$$2e_x^T PE_i d \leq \varepsilon_2 e_x^T e_x + \rho^2 \varepsilon_2^{-1}\left\|PE_i\right\|^2 \quad (20)$$

$$2e_x^T P\varphi_i = \rho^2 \varepsilon_2^{-1}\frac{\left\|PE_i\right\|^2}{e_y^T e_y} e_y^T PP^{-1}\sum_{j=1}^{r} h_j C_j^T e_y$$
$$2e_x^T P\varphi_i = \rho^2 \varepsilon_2^{-1}\left\|PE_i\right\|^2 \quad (21)$$

From (20) and (21) we deduce that :

$$\dot{V} \leq \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z)h_j(z)e_x^T(\bar{A}_{ij}^T P + P\bar{A}_{ij} + \varepsilon_1^{-1}P^2 + \varepsilon_2 I)e_x \quad (22)$$

We assume that, the estimation error is insensitive to the uncertainties modeled by the term $E_i d(t)$, and converges towards zero if the relation (22) holds.

V. Simulation and Results

For a reliable simulation of the anaerobic digester process, the nonlinear model 1 is used as a virtual system to generate the data. However, it is not very easy to select either the input or the output variables for the process.

In this work, $[D, S_1^{in}, S_2^{in}, C_{TI}^{in}, Z^{in}]$ are selected as inputs where $u = D$ is the controlled input and $d = [S_1^{in}, S_2^{in}, C_{TI}^{in}, Z^{in}]$ is the disturbances vector $[S_1, S_2, Z, Q_{CH_4}]$ are selected as outputs.

Time evolution of the measured and unmeasured inputs over 40 days are shown in figure 2, and the biogas output in figure 3.

The nonlinear model is approximated by T-S fuzzy multiple model with eight state space sub-systems for wich local linear fuzzy observers are constructed, by taking $[D, Q_{CH_4}]$ as premise variables and they are fuzzified as shown in figure 4.

Figure 5 illustrates the high performances of Takagi-sugeno modelling for anarerobic digestion plant. Comparing of measured and estimated states in figure 6, we assume that FSMO can occure very well the real states in presence of unknwon inputs.

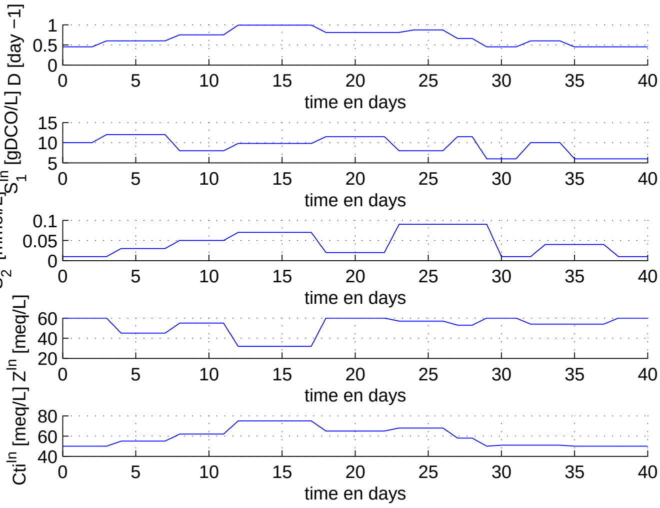

Figure 2. Data sequences of the process.

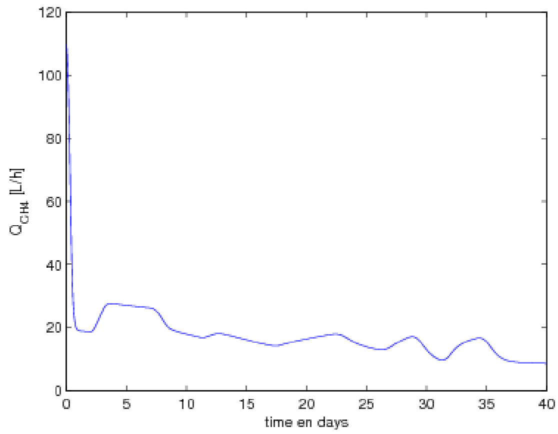

Figure 3. Methane flow rate

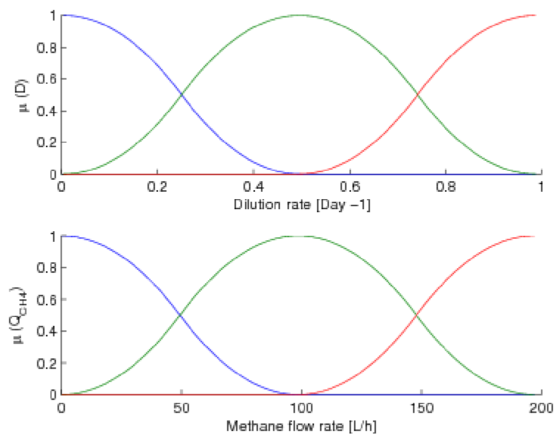

Figure 4. Membership functions for D and Q_{CH4}

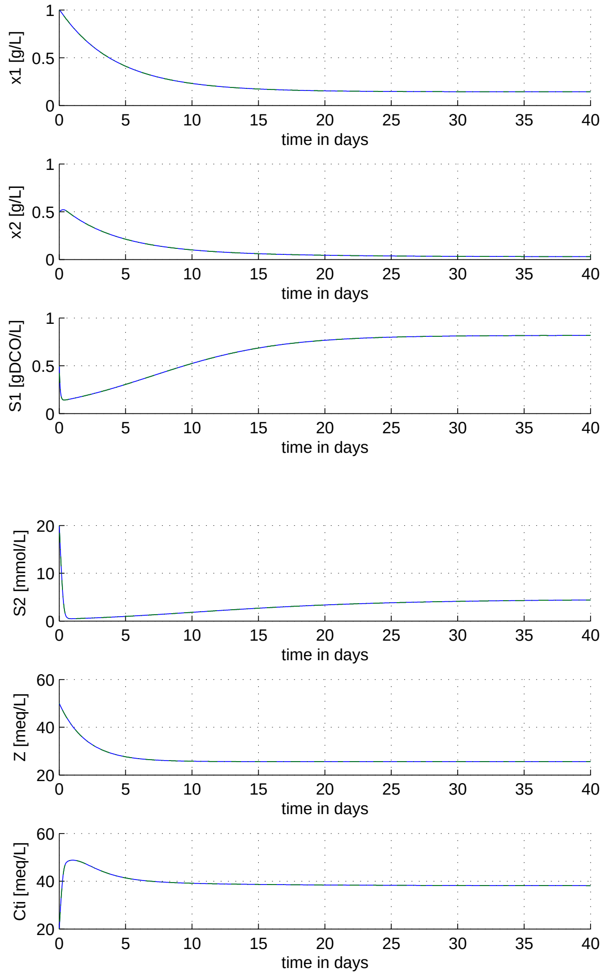

Figure 5. Performances of Takagi-sugeno modelling for anarerobic digestion system

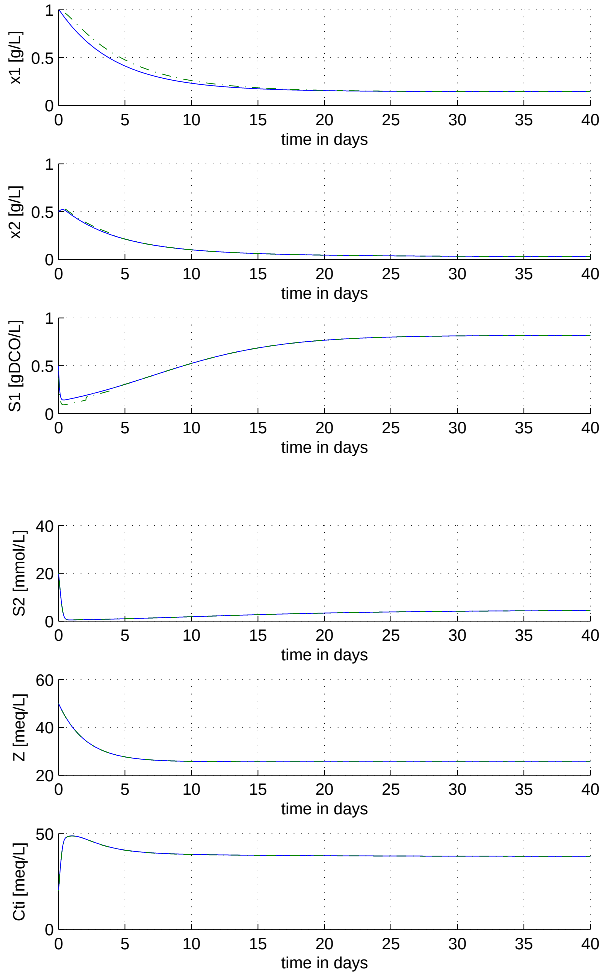

Figure 6. State estimation for ADP using $FSMO$

VI. Conclusion

In this paper, Takagi-Sugeno fuzzy sliding mode observers can be constructed for anaerobic digestion system described by unknown input Takagi-Sugeno fuzzy model representation. Such local FSMO can have guaranteed stability and performance. Then, the whole FSMO will converge asymptotically to the real system states. The design methodology demands the solution of linear matrix inequalities LMIs problem, using an efficient numerical methods. The simulation results prove the effectiveness of the proposed fuzzy sliding mode observers.

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